1 Preamble

According to Lindström 1966 a quantifier is a functor which assigns to each non-empty domain a relation among relations which is closed under isomorphisms. A simple instance of this notion is given by the quantifier 'more than half of the', which for each domain $E$ gives the relation between sets $A, B \subseteq E$ defined by:

$$|\{a \in A : a \in B\}| > |\{a \in A : a \notin B\}|$$

In the present collection of articles the authors investigate several aspects of such quantifiers, also of quantifiers with relational arguments.

This introduction presents some basic insights and techniques of quantification theory. After a brief history, we pay attention to application of the theory in linguistics, and then to its more logical features. The linguistic topics include: denotational constraints, behaviour in certain linguistic contexts, and polyadic forms of quantification. On the logical side, we discuss metaproperties of weak and of 'real' quantifier logics. In particular, we concentrate on the tableau method for weak quantifier logics, and on decidability results.

It is impossible to write an introduction to this field which does not overlap with the comprehensive overviews in Westerståhl 1989, van Eijck 1991, Keenan and Westerståhl 1995, Westerståhl 1995, and the reader is encouraged to study some of these as well. For surveys of recent work we recommend van Benthem and Westerståhl 1994, Westerståhl 1995, Keenan and Westerståhl 1995.
2 A Trace of History

2.1 Aristotle

Aristotle was already aware that quantifiers play a key role in the process of making inferences, so ever since Aristotle’s day quantification is a central topic in logic. In his theory of the syllogism, Aristotle studied the following inferential pattern:

\[
\begin{align*}
\text{Quantifier}_1 & \text{ Restriction}_1 & \text{Body}_1 \\
\text{Quantifier}_2 & \text{ Restriction}_2 & \text{Body}_2 \\
\text{Quantifier}_3 & \text{ Restriction}_3 & \text{Body}_3
\end{align*}
\]

As an example we give the valid syllogism FESTINO:

- No A are B
- Some C are B
- Some C are not A

Syllogistic theory focusses on the quantifiers in the so-called Square of Opposition given in figure 1. The quantifiers in the square express relations

\[
\begin{align*}
\text{All A are B} & \quad \text{No A are B} \\
\text{Some A are B} & \quad \text{Not all A are B}
\end{align*}
\]

between a first and a second argument (the restriction and the body), where both arguments denote sets of entities taken from some domain of discourse. In the square the quantified expressions are related across the diagonals by external (sentential) negation, and across the horizontal edges by internal (or verb phrase) negation. It follows that the relation across the vertical edges of the square is that of internal plus external negation; this is the relation of quantifier duality.

Aristotle interprets his quantifiers with existential import: All A are B and No A are B are taken to imply that there are A. Under this assumption, the quantified expressions at the top edge of the square imply those immediately below them. The universal affirmative quantifier all implies the individual affirmative some and the universal negative no implies the individual negative not all. Existential import also makes that the two quantified expressions on the top edge of the square cannot both be true; these expressions are called contraries. For the same reason, the two quantified expressions on the bottom edge cannot both be false: they are so-called subcontraries. See van Benthem 1984, and van Eijck 1985 for information on the connection between syllogistics and generalized quantifier theory.

Aristotle’s syllogistics is quite an impressive theory of quantification, but it has some shortcomings. In the first place, quantifier combinations are not treated: only one quantifier per sentence is allowed. Secondly, ‘non-standard quantifiers’ such as most, half of the, at least five, … are not covered. A minor additional flaw is the assumption of existential presupposition. In mathematical reasoning, and sometimes also in everyday reasoning, one wants to be able to assert universally quantified statements without assuming existence. Cf. De Jong and Verkuyl 1985 for a discussion.

2.2 Frege, Peirce

Independently, Gottlob Frege and Charles Sanders Peirce invented predicate logic, and the theory of quantification that goes with it. Their approaches are based on the introduction of individual variables bound by the quantifiers ∀ (‘for all’) and ∃ (‘there exists’). These are the so-called standard quantifiers. This account of quantification removes the first of the three defects of the Aristotelian theory. Quantifiers with their associated variables can combine with arbitrarily complex predicate logical formulae to form new predicate logical formulae, and a formula may contain an arbitrary number of quantifiers.

The quantifiers ∀ and ∃ are interdefinable with the help of negation: Something is rotten means the same as It is not so for every x that x is not rotten, and Everybody is happy means the same as It is not so that there is a person x who is not happy. More formally: ∃x Ax is true if and only if ¬∀x¬Ax is true, and ∀x Ax is true if and only if ¬∃x¬Ax is true.

On this view, the Square of Opposition looks like figure 2. Apart from the existential presuppositions, the Aristotelian quantifiers from the Square of Opposition can be expressed in terms of the Fregean standard quantifiers, as follows (with ~ for ‘translates as’):

\[
\begin{align*}
\text{All A are B} & \quad \sim \forall x (Ax \rightarrow Bx) \\
\text{Some A is/are B} & \quad \sim \exists x (Ax \land Bx) \\
\text{No A is B} & \quad \sim \forall x (Ax \rightarrow \neg Bx) \\
\text{Not all A are B} & \quad \sim \exists x (Ax \land \neg Bx)
\end{align*}
\]
3 Linguistic Issues

On the linguistic side, quantifier theory has several things to offer. In the first place, it provides the means for a compositional semantics for natural language sentences. Secondly, it gives insight into which natural language determiners are realized among the many possibilities. Thirdly, it describes and sometimes even explains the behaviour of noun phrases in particular linguistic contexts. Finally, it allows us to give the truth-conditions for sentences which are hard to understand otherwise. In this section, we give examples of all these features of the theory.

3.1 Misleading Form vs. Compositionality

In a previous era of natural language semantics, initiated by Russell and Wittgenstein, the familiar formulation of first-order predicate logic was considered the one and only tool for semantic analysis. As a consequence, quantified noun phrases were commonly regarded as systematically misleading expressions. This is called the misleading form thesis.

Consider the translation of example (1).

(1) Every farmer bought a cow.
\[ \forall x (F x \to \exists y (C y \land B x y)) \] .

Example (1) illustrates that first-order logic has no difficulty with quantifier combinations. But observe that the translation does not contain phrases corresponding to the noun phrases every farmer or a cow. Given a natural language sentence and its translation into first-order logic, as presented in the familiar way, it is impossible to pinpoint the subexpression in the translation that gives the meaning of a particular noun phrase in the original. In the translation into first-order logic, the noun phrases have been syntactically eliminated, so to speak. This illustrates that the natural language syntax of quantified expressions does not correspond to this predicate logical syntax. In natural language, quantified noun phrases are separate constituents, but they evaporate during the process of translation into first-order logic.

To demonstrate that in the relational perspective on quantification the suggestion of misleading form disappears, we consider two simple example sentences. We show that a representation language with generalized quantifier expressions (expressions denoting two place relations between sets) and a notation for lambda abstraction is well suited for the compositional analysis of natural language sentences with quantified noun phrases.

First consider example (2).

(2) Every woman smiled.

This sentence is composed of a noun phrase every woman, composed in turn of a determiner every and a noun woman, and a verb phrase smiled.
The determiner *every* translates into an expression *every* denoting a function from properties to a function from properties to truth values. More precisely, *every* denotes the function mapping property $P$ to (the characteristic function of) the set of all properties having $P$ as a subset. The noun *woman* translates into $\lambda x.Wx$, the verb phrase *smiled* into $\lambda y.Sy$, the noun phrase *every woman* into $\lambda x.Wx$, and, finally, the whole sentence into the expression $(\lambda x.Wx)(\lambda y.Sy)$. The reader is urged to check that this expression yields true in case the property of being a woman is included in the property of smiling, false otherwise.

As a second example we consider example (1) again, also to see how quantifier combinations are dealt with compositionally. The trick is finding the right translation for the transitive verb. This turns out to be the lambda expression $\lambda X\lambda y.X(\lambda z.Byz)$, where $X$ is a variable over noun phrase type expressions. The verb translation is of the right type to take the object noun phrase translation as its argument; this gives translation (3a) for the verb phrase, which reduces to (3b).

\[(3)\text{ a. } \lambda X\lambda y.X(\lambda z.Byz)(a(\lambda u.Cu)) \]
\[(3)\text{ b. } \lambda y.(a(\lambda u.Cu))(\lambda z.Byz) \]

Here $a$ denotes the function which maps every property $P$ to (the characteristic function of) the set of all properties having a non-empty overlap with $P$. Feeding (3b) as argument to the expression $(\lambda x.Fx)$, the translation of the subject, one gets (4) as translation for the whole sentence.

\[(4) \quad (\lambda x.Fx)(\lambda y.(a(\lambda u.Cu))(\lambda z.Byz)) \]

This translation can still be simplified somewhat, by writing $F$ and $C$ for the property denoting expressions $\lambda x.Fx$ and $\lambda u.Cu$, respectively.

\[(5) \quad (\lambda y.(a(\lambda u.Cu))(\lambda z.Byz)) \]

The compositional semantic analysis of natural language sentences involving quantifiers is the reverse of the process of compositional synthesis demonstrated here.

### 3.2 Quantifier Constraints

The sentence *All men walk* is true in a given model if and only if the relation of inclusion holds between the set of men in the model and the set of walkers in the model. Abstracting from the domain of discourse, we can say that determiner interpretations (henceforth: determiners) pick out binary relations on sets of individuals, on arbitrary universes (or: domains of discourse) $E$. Notation: $D_{E}AB$. We call $A$ the restriction of the quantifier and $B$ its body. If $D_{E}AB$ is the translation of a simple sentence consisting of a quantified noun phrase with an intransitive verb phrase then the noun denotation is the restriction and the verb phrase denotation the body. See figure 3 for a graphical representation.

![Figure 3: Quantifiers as Relations](image)

A simple binary quantifier $D$ on a domain $E$ is a relation between subsets of $E$:

$$D_{E} \in \mathcal{P}(\mathcal{P}(E) \times \mathcal{P}(E))$$

The trivial quantifiers are $\top_{E}$ and $\bot_{E}$, which hold of all and of no pairs of sets, respectively.

Not all elements in $\mathcal{P}(\mathcal{P}(E) \times \mathcal{P}(E))$ serve as natural language determiner denotations. In fact, one of the first insights provided by quantification theory is that such determiners have to satisfy some constraints. A first requirement is extension:

**EXT** For all $A, B \subseteq E \subseteq E'$: $D_{E}AB \Leftrightarrow D_{E'}AB$.

A relation observing EXT is stable under growth of the universe. So, given sets $A$ and $B$, only the objects in the minimal universe $A \cup B$ matter. See figure 4.

![Figure 4: The Effect of EXT](image)
An example of a natural language determiner which does not satisfy EXT is \textit{many} in the sense of \textit{relatively many}:

\[ |A \cap B| > 0.5 \times |E| \]

In this sense \textit{many} crucially depends on \( E \).

A second requirement for quantifier relations is \textit{conservativity}:

\textbf{CONS} For all \( A, B \subseteq E \): \( D_E AB \Leftrightarrow D_E AA \cap B \).

In the context of EXT, this property expresses that the first argument of a relation (the interpretation of the noun) plays a crucial rôle: it sets the stage, in the sense that everything outside the extension of the first argument is irrelevant.

There are a few noun phrase determiners which do not satisfy CONS. One example is \textit{only} as in (6).

\textbf{(6)} Only men came to the party.

This example is true in a situation where all partygoers were men; \textit{only} denotes the superset relation: \( \supseteq \). Starting out from a situation like this, and adding some women to the partygoers will make (6) false. This shows non-conservativity. All is still well if it can be argued that noun phrases starting with \textit{only}, \textit{mostly} or \textit{mainly} (two other sources of non-conservativity) are exceptional syntactically, in the sense that these noun phrase prefixes are not really determiners. In the case of \textit{only}, it could be argued that \textit{only men} has structure [np [mod only] [np men]], with \textit{only} not a determiner but a noun phrase modifier, just as in (7).

\textbf{(7)} Only John came to the party.

See De Maey, this volume, for more information concerning \textit{only}.

Despite the small number of counterexamples, separating out the determiners satisfying CONS and EXT is important, for the combination of EXT and CONS is equivalent to the principle UNIV:

\textbf{UNIV} For all \( A, B \subseteq E \): \( D_E AB \Leftrightarrow D_A AA \cap B \).

The right-hand-side of this equivalence shows that for UNIV \( D \) the truth of \( D_E AB \) only depends on the sets \( A \) and \( A \cap B \); or, on finite domains equivalently, on the sets \( A - B \) and \( A \cap B \) (respectively, the things which are \( A \) but not \( B \), and the things which are both \( A \) and \( B \)). See figure 5.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{The Combined Effect of EXT and CONS}
\end{figure}

A UNIV determiner still lacks the property that it is only sensitive to the cardinalities of the sets \( A - B \) and \( A \cap B \). But such insensitivity is something one would surely expect from a quantifier. The relational perspective suggests a very natural way of distinguishing between expressions of quantity and other relations. Quantifier relations satisfy the following condition of \textit{isomorphy}, formulated in terms of bijections.

\textbf{ISOM} If \( f \) is a bijection from \( E \) to \( E' \), then \( D_E AB \Leftrightarrow D_E f[A][f[B]. \)

Here \( f[A] \), the image of \( A \) under \( f \), is the set of all things which are \( f \)-values of things in \( A \). If \( D \) satisfies EXT, CONS and ISOM, it turns out that the truth of \( DAB \) depends only on the cardinal numbers \( |A - B| \) and \( |A \cap B| \). See figure 6 for the combined effect of these three conditions, and section 4.3 for a proof. By definition a quantifier is a relation \( D \) satisfying EXT, CONS and ISOM.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure6.png}
\caption{The Combined Effect of EXT, CONS, ISOM}
\end{figure}

The semantic effect of a quantifier \( DAB \) can always be described in terms of the properties of the numbers \( |A - B| \) and \( |A \cap B| \). All \( A \) are \( B \) is true if and only if the number of things which are \( A \) and not \( B \) is 0. Some \( A \) is \( B \) is true if and only if the number of things that are both \( A \) and \( B \) is at least 1. Most \( A \) are \( B \) is true if and only if the number of things that are both \( A \) and \( B \) exceeds the number of things that are \( A \) and not \( B \). Some further examples are in section 4.3.1.

\subsection{3.3 Distribution and Logical Behaviour}

We now turn to the distribution of noun phrases in existential sentences, and to their logical behaviour in naked infinitive perception reports.

\subsubsection{3.3.1 Existential Sentences}

Not all noun phrases may occur in so-called existential sentences. For instance, the noun phrases in (8a) are acceptable while those in (8b) are not.

\begin{itemize}
  \item[(8)] a. There are two/some/no students at the party
  \item[(8)] b. *There are all/the/not all students at the party
\end{itemize}
In Miltsark 1977 the noun phrases which are allowed in these contexts are categorized as the weak ones, while those who are not he calls strong. (Weak determiners are also called 'indefinite'.) Barwise and Cooper 1981 give these notions semantical content by means of familiar relational properties:

**Definition 1** A determiner \( D \) is positive strong iff \( D \) is reflexive:

\[ \forall X . DXX \]

\( D \) is negative strong iff \( D \) is irreflexive:

\[ \forall X . \neg DXX \]

\( D \) is strong iff \( D \) is either positive or negative strong. \( D \) is weak iff \( Q \) is not strong.

It accords nicely with Miltsark's taxonomy that two, some, and no are weak in this semantic sense. Typical examples of strong quantifiers are all and not all. Indeed, the positive strong conservative determiners extend all, while the negative strong ones are part of not all. E.g., if \( A \subseteq B \), then by conservativity and reflexivity: \( DAA, DAA \cap B, DAB \). So, \( all \subseteq D \).

The proposal of Barwise and Cooper offers an explanation of why strong noun phrases cannot occur in existential sentences. A sentence of the form (9a) can be said to be true (relative to a domain \( E \)) iff (9b) is true.

\[ (9) \]

a. There are \( \mu P.DET.N \)

b. \( [DET]_E[N]\in E \)

In case the determiner is conservative this would mean that (9a) is true iff \( [DET]_E[N]\in[N] \). So, in an existential sentence positive strong determiners yield logical truths, while negative strong determiners yield contradictions. But a simple existential sentence is contingent: sometimes true, sometimes false.

A problem for the above proposal is that all proportional determiners \( n/\% \) of the are semantically weak:

\[ n/\% \text{ of the } AB \Leftrightarrow |A \cap B| \geq n/100 \times |A| \]

but none of them is acceptable in an existential sentence.

\[ (10) \]

a. There are five percent of the students at the party

These counterexamples suggest to redefine the weak determiners as the symmetric ones. This notion of weakness is more restrictive than that of Barwise and Cooper. Assuming conservativity, it can be seen that the only strong symmetric determiners are the trivial \( \top \) and \( \bot \). For \( DAB \):

- \( DAB \) iff (conservativity) \( DAA \cap B \) iff (symmetry) \( DA \cap BA \) iff (conservativity) \( DA \cap BA \)

- \( DAB \) and the latter is always true or always false for strong determiners. Therefore, non-trivial symmetric determiners are weak.

Identifying weakness with symmetry squares well with attempts to generalize the notion to determiners of higher arities, such as the comparative determiners more than, as many as, fewer than, which are 3-place determiners. These all occur felicitously in existential sentences.

\[ (11) \]

a. as many as \( ABC \Leftrightarrow |A \cap B| = |A \cap C| \)

b. There are as many students as teachers at the party

A proposal for a generalization of weakness by Keenan 1987a is to identify the indefinite (or weak) determiners with the intersective ones (the following definition is equivalent to Keenan's):

**Definition 2** An \( n \)-place determiner \( D \) is intersective iff \( D \) is conservative and \( DA_1 \ldots A_n.B \Leftrightarrow DA_1 \cap B \ldots A_n \cap BB \).

Keenan's formalization of weakness is as general as one would like it to be. To see that it generalizes the notion of symmetry to the \( n \)-ary case, note that binary conservative determiners are intersective iff they are symmetric. For assume that \( D \) is symmetric. Then \( DAB \) iff (conservativity) \( DAA \cap B \) iff (symmetry) \( DA \cap BA \) iff (conservativity) \( DA \cap BA \) iff (conservativity) \( DA \cap BB \). Conversely, if we assume that \( D \) is intersective, then \( DAB \) iff (intersectivity) \( DA \cap BB \) iff (conservativity) \( DA \cap BA \) iff (intersectivity) \( DBA \cap B \) iff (conservativity) \( DBA \). It follows that Keenan's proposal is empirically more adequate than that of Barwise and Cooper: the proportional determiners are not intersective but numerals and comparatives are.

### 3.3.2 Naked Infinitive Perception Reports

Studying quantifiers in a partial setting is necessary, among other things, to be able to deal with the semantics of naked infinitive perception reports (Barwise 1981, Barwise and Perry 1983, Higginbotham 1983, Kamp 1984, Asher and Bonevac 1987, Asher and Bonevac 1989). Examples of such perception reports are in (12a-c), where the complement of the perception verb is unconjugated.

\[ (12) \]

a. I saw some bears prepare sandwiches.

b. I saw no bears prepare sandwiches.

c. I saw two bears prepare sandwiches.

A key feature of the semantics of naked infinitives is the fact that sentences like (13) have a reading which does not imply variants where the perception complement is replaced by a classically equivalent complement, as in (13a-b).

\[ (13) \]

a. I saw John help Mary.

b. I saw John help Mary and help Bill or not help Bill.

In a classical framework the complements in (13a,b) are logically equivalent, so the semantic distinction between the two examples gets lost. To
Examples of MON\textsuperscript{\dag} determiners are: all, some, at least two, while Not all and no are MON\textsubscript{\dag} determiners. Some and not all are \textsuperscript{\dag}MON\textsuperscript{\dag}; All and no are \textsuperscript{\dag}MON. There are also non-monotonic determiners; e.g., exactly two and an even number of are neither MON\textsuperscript{\dag} nor MON\textsubscript{\dag}.

The characterization of veridical complements in terms of MON\textsuperscript{\dag} corresponds nicely with the logical implication relations between (12a,c). An explanation for this behaviour is that the verb see restricts the extensions of verbal elements within its scope, and these verbal elements occur within the right-hand side argument of the determiners within the complement of see. Still one may wonder whether left monotonicity is as important. This does not seem to be the case. E.g., (16a) has (16b) as a consequence, and (16c) (16d). But every is \textsuperscript{\dag}MON and most is not left monotone.

(16) a. At the party, John saw every student leave
    b. At the party, every student left
    c. At the party, John saw most students leave
    d. At the party, most students left

Naked infinitive perception reports make plain that it is of considerable interest to extend quantifier theory to cover the many-valued case. As Van Eijck, this volume, shows, the extension involves providing suitable extensions of the principles EXT, CONS and ISOM, among many other things. See also Muskens 1989, and Langholm 1988.

3.4 Polyadic Quantification

In section 3.1, we have seen that a transitive sentence such as (17a) can be interpreted by iterating the noun phrases denotations as in (17b).

(17) a. Every farmer bought a cow
    b. every \((\lambda x. Fx)(\lambda y. (\lambda z. Cz)(\lambda z. Byz))\)

Due to the work of Higginbotham and May 1981, Keenan 1987b, 1992, van Benthem 1989, Westerståhl 1994a it has become more and more apparent that not all sentence meanings can be obtained in the iterative way. In this section, we give some examples of such non-iterative forms of quantification; see Keenan 1987b, this volume, and Ben-Shalom 1994 for more examples.

3.4.1 Cumulative Quantification

Cumulation is perhaps the simplest form of non-iterative quantification, and is first observed by Scha 1981. Sentence (18a) may be true if there are ten firms which each own twenty computers, as in (18b).

(18) a. Ten firms own twenty computers
    b. \(|\{f \in F : \{c \in C : Ofc\} = 20\}| = 10\)
    c. \(|\{f \in F : \exists c \in C Ofc\}| = 10 \& \{|c \in C : \exists f \in F Ofc\}| = 20\)
But (18a) can also be used to state that ten firms own computers and that twenty computers are owned by firms. This is (18c), which leaves the numerals ten and twenty outside each others scope.

3.4.2 Branching Quantification
Branching, like cumulation, is a form of quantification where the scopes of the noun phrases remain independent. Following up on Hintikka 1974, Barwise 1979 studies sentences like (19a-c), which require branching of non-first-order quantifiers.

(19) a. Most men and most women like each other
b. Few men and few women like each other
c. Four men and two women like each other

The meanings of (19a-c) are respectively given by (20a-c):

(20) a. \( \exists x y [M X \land M Y \land (X \land Y) \land (X \land Y) \in R \land (M \land M )] \)
b. \( \exists x y [F X \land F Y \land R \land (M \land M ) \in (X \land Y) \land (M \land M )] \)
c. \( \exists x y [12 M X \land (X \land X \land \Delta X) \land (M \land M ) = R \land (M \land M )] \)

Notice that these meanings are not uniform across all determiners. The recipe in (20a) is intended for MON\(\uparrow\) and that in (20b) for MON\(\downarrow\) determiners (cf. Barwise 1979). Van Benthem introduced (20c) for non-monotone determiners. Westerstahl 1987 has a first proposal for a more general definition of branching. Cf. also Sher 1990, or Liu and Spaan, this volume.

Cumulation and branching are closely related; in each of the above cases branching implies cumulation. As Westerstahl 1987 observes, they are even equivalent for MON\(\downarrow\) determiners. A further observation is that the branching of non-monotone determiners is equivalent to cumulation plus the statement that \( R \land A \land B \) is a cartesian product. The MON\(\uparrow\) case, however, is quite different. Then, cumulation and branching are still equivalent on cartesian products, but there is no statement in first-order logic with the MON\(\uparrow\) determiners added which defines the branching reading.

3.4.3 Reciprocals

Another form of polyadic quantification, which appears similar to branching, is used to give one of the several meanings of reciprocals. For instance, the prominent readings of (21a-c) are formally represented by means of (variants of) the so-called Ramsey quantifier in (22a-c). Cf. Dalrymple et al. 1994, Keenan and Westerstahl 1995.

(21) a. Most men like each other
b. Few men like each other
c. Twelve men like each other

(22) a. \( \exists x y [M X \land (X \land X \land X) \land M \land M \in R] \)
b. \( \exists x y [F X \land F Y \land M \land M \in (X \land X \land X) \land M \land M ] \)
c. \( \exists x y [12 M X \land (X \land X \land \Delta X) \land M \land M = R \land M \land M ] \)

Here, \( \Delta(X) := \{ (d, d) : d \in X \} \). The use of \( \Delta \) ensures that in none of the cases the relevant men have to like themselves.

3.4.4 Resumptive Quantification

Explicit quantification can also be found outside noun phrases, in particular in adverbial modifiers. English has explicit adverbs of quantification which run over locations (everywhere, somewhere, nowhere), over periods of time (always, sometimes, never), and over states of affairs (necessarily, possibly, impossibly). Like noun phrase quantifiers, these standard adverbial quantifiers have non standard cousins: often, seldom, at least five times, more than once, exactly twice, and so on.

Adverbial quantifiers behave very much like noun phrase quantifiers, the main difference being their different domain of quantification. An exact specification of the domain of quantification can be difficult. Contextual information may be needed to determine whether an adverbial quantifier ranges over periods of time, events, or occasions, and to determine the 'granularity' of the domain of quantification. E.g., the fact that the temporal adverb in (23) ranges over days has to be inferred from the overall meaning of the sentence.

(23) Dinner is always served at six p.m. here

A very influential proposal for the treatment of adverbs of quantification is that of Lewis 1975. He argues that the adverb in (24a) quantifies over cases, which are identified with the tuples in a the relation \( \lambda X. M x \land W y \land L x y \).

(24) a. Mostly, a man loves a woman
b. \( M (a x y. M x \land W y \land L x y) \)

Formally, this means that the quantifier most as it applies to sets has to be lifted so that it can take relational arguments. This can be done as follows (cf. Westerstahl 1989):

\[ M^n_E R^n \leftrightarrow M_{E^n} R^n \]

By this definition \( M^n \) is a property of \( n \) place relations, since

\[ M_{E^n} \subseteq \bigwedge(E \times \cdots \times E) \]

\[ n \text{ times} \]

It should be observed that the relational view on adverbs of quantification assumes that the indefinites an \( N \) are open expressions of form \( N x \). In particular, they are not quantifiers. This position is developed further in the discourse representation theory of Kamp 1981 and Heim 1982. Ladusaw, this volume, applies this approach to negative polarity items. Cf. also De Swart, this volume, for the issue of temporal expressions and quantification. In this volume, discussions of other linguistic issues can be found in van den Berg (quantifiers and anaphora), Hoeksema (exception phrases),