Adapted from problems from Hopcroft et al. (2001).

**Problem 1.** (2 points) Draw state diagrams for DFSAs for each of the following languages:
(a) (1 of 2 points) The set of all strings over \( \{a, b\} \) such that any sequence of two \( a \)'s is immediately followed by three \( b \)'s.
(a) (1 of 2 points) The set of all strings over \( \{a, b\} \) such that the number of \( a \)'s is divisible by two and the number of \( b \)'s is divisible by 3.

**Problem 2.** (3 points) Prove that language of the DFSA given by the following state table is the set of all strings over \( \{a, b\} \) containing exactly one \( a \). States in bold are final states; \( q_0 \) is the initial state.

|     | a   | b
|-----|-----|-----
| \( q_0 \) | \( q_1 \) | \( q_0 \)
| \( q_1 \) | \( q_2 \) | \( q_1 \)
| \( q_2 \) | \( q_2 \) | \( q_2 \)

**Problem 3.** (2 points) Prove that \( w_1w_2 \)—that is, the language of all strings that can be decomposed into exactly two repetitions of a single string—is not a regular language for any alphabet containing at least two symbols.

**Problem 4.** (1 point) Consider the language from Problem 3 in the case where the alphabet has one symbol. Prove that it is regular.

**Problem 5.** (2 points) Prove that the set of all strings \( wa^n \) over \( \{a, b\} \) where \( |w| = n \) is not a regular language.

**Problem 6.** (2 points)
(a) (1 of 2 points) Draw a state diagram for an NFSA for \( a(b+a)^k a \), where \( k \) is even.
(b) (1 of 2 points) Give the DFSA constructed from this NFSA by the subset construction (leave out inaccessible states).

**Problem 7.** (3 points) The interleaving of two languages \( L_1 \) and \( L_2 \) is the set of all strings \( w = w_1 w_2 \cdots w_{1k} w_{2k} \), where \( w_1 = w_{11} w_{12} \cdots w_{1k} \) is a string of \( L_1 \), and \( w_2 = w_{21} w_{22} \cdots w_{2k} \) is a string of \( L_2 \). In the case where \( w = \epsilon \), \( w \) is an element of the interleaving exactly when \( \epsilon \in L_1 \) and \( \epsilon \in L_2 \). Show that the regular languages are closed under interleaving. **Hint:** Your answer may well take up several pages. Even if you cannot get all of the details down precisely, make sure you get the form of the argument right, for partial credit.

**References**