Some important regular relations

Definition: Given a language $L$, $Id(L)$ is the identity relation on $L$, i.e., the relation which is defined only for $L^2$ and which is such that

$$Id(L)[s \rightarrow s'] \iff s = s'.$$

Clearly $Id(L)$ is a constant on $\Sigma^*.$

Trivial to show that if $L$ is a regular language, $Id(L)$ is a regular relation just replace $x$ with $xx$ or $xx$ are relabeled.

Definition: Given $L_1$, $L_2$, $Prod(L_1, L_2)$ is the relation $R$ s.t. $R[s \rightarrow t] \iff s \in L_1$ and $t \in L_2.$

Claim: If $L_1$, $L_2$ are regular, then $Prod(L_1, L_2)$ is a regular relation.

Proof: Let the carrier be the usual product monoid, but with $\Sigma_1 \times \Sigma_2$ as alphabet

$$s = (s_1(a_1) \times s_2(a_2)).$$

Easy to show that

$$s_1(a_1) \times s_2(a_2)$$

which is really
Case of the proof that rules are cycles:

We start with the following relation:

$A \rightarrow B$

Replace by $(\text{Id} (\Sigma^*) \circ \text{Opt}(A \rightarrow B))^*$

We will the replacement of $A \rightarrow B$ optimal because it will turn out to be a simpler place to start.

i.e. $\rightarrow \text{Opt}(R)(a \rightarrow b)$ if $R(a \rightarrow b)$ or $a = b = \varepsilon$ can be satisfied.

But before we can do this, we need to show that construction and $*$ are going to preserve regularity.
Theorem: Suppose $R_1$ and $R_2$ are regular relations. Define $R_1 R_2$ be the relation induced by $E(R_1) E(R_2)$, the retension of the two relations.

Proof:

First, the two regular relations put a transition between the accepting states of $R_1$ and the initial state of $R_2$.

Define the new accepting states to be the accepting states of $R_2$.

Theorem: Suppose $R$ is a regular relation. Then $R^*$, the relation induced by $E(R)^*$, is regular.

Proof:

Define the initial state as an accepting state. Add a transition from any final state to the initial state.
Let's see a actual model for the replace relation.

We start with

\[
\text{Prod}(a, b)
\]

So we construct a PSN for

\[
\text{Pres}
\]

and a PSN for

\[
\text{Ids}
\]

\[
\rightarrow 0 \quad \rightarrow 0\quad \rightarrow 0 \quad \rightarrow 0
\]

We cross these

\[
\begin{array}{c}
\text{Prod}(a, b) \\
\xrightarrow{a \ast b}
\end{array}
\]

\[
\begin{array}{c}
\text{Pres} \\
\xrightarrow{a \ast b}
\end{array}
\]

\[
\begin{array}{c}
\text{Ids} \\
\xrightarrow{a \ast b}
\end{array}
\]

\[
\Rightarrow \text{concatenate}
\]

\[
\rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow
\]

\[
\text{Pres}
\]

\[
\text{Ids}
\]
we can't do this.

\[ \text{Id}(X) \cdot \text{opt}(\text{Id}(V) \cdot \text{Res}(a, b) \cdot \text{Id}(W)) \]

because the V's can't overlap in that order and 12 content

idea: preprocessing thing:

> here's where the right contact is

> here's where the right contact is

> V < a < V < a > V <

Can have brackets inside...?

here is kris' running example:

\[ N \rightarrow m \rightarrow \text{[table]} \]

\[ \text{[...]} \]

hit at here