A few more closure properties of RLs

February 13, 2012

A few more closure properties

Recall that the regular languages are closed under intersection.

**Theorem 1.** If $L$ is a regular language, $\bar{L} = \Sigma^* - L$ is also regular.

**Proof.** Construct a DFSA $A$ for $L$. Now construct a new DFSA $A'$ identical to $A$ but with final states $Q - F$. We claim that $L(A') = \bar{L}$: given that $w \in L \iff \hat{\delta}(q_0, w) \in F$, $w \notin L \iff \hat{\delta}(q_0, w) \notin F \iff \hat{\delta}(q_0, w) \in Q - F$. □

**Theorem 2.** If $L_1$ and $L_2$ are regular languages, $L_1 \cup L_2$ is also regular.

**Proof.** $L_1 \cup L_2 = \overline{\overline{L_1 \cap L_2}}$. Since the regular languages are closed under intersection and complementation, the theorem holds. □

**Theorem 3.** If $L$ is a regular language, $L^R$, the set of all strings $w$ such that $w^R$, the reversal of $w$, is in $L$, is also regular.

**Proof.** (Idea). Remember that we briefly mentioned the ability to add $\epsilon$-transitions to an NFSA. The idea is to reverse $\delta$, so that, if $\delta(q, a) = p$, now $\delta'(p, a) = q$. We make $q_0$ the sole final state of a such an NFSA, and add a new start state $q'_0$ which has an $\epsilon$-transition to all states that were previously final states. □