Chapter 8

Generalized Quantifier theory

In this chapter, we discuss the interpretation of NPs and determiners in terms of generalized quantifier theory. We further discuss semantic properties of determiners and NPs, which play a role in the explanation of such phenomena as existential constructions, conjunction/disjunction reduction, partitives, and negative polarity items.

8.1 NPs and determiners

In the previous chapter, we pointed out that expressions like *Socrates*, *every student* and *most linguists* are syntactically characterized as NPs. If we treat the NP as a syntactic unit, a compositional theory of meaning requires that it also be a semantic unit. The question then arises what kind of denotation we can associate with NPs. In our introduction to first-order predicate logic, we argued that we can combine predicates and arguments by predication. If we combine a one-place predicate like *snore* and a proper name like *Jane* as in (1), we predicate the property of snoring of Jane, and translate the sentence as in (1a):
(1) Jane snores.
   a. Snore(j)
   b. \[ \{ \text{Snore}(j) \} = 1 \text{ iff } [j] \in [\text{Snore}] \]
   c. \[ \text{Snore} = \{ x \in U | \text{Snore}(x) \} \]

We interpret the predication relation in (1a) as set-membership and claim that the individual referred to by the constant \( j \) has to be a member of the interpretation of the predicate in order for the sentence to be true (1b). Given that the one-place predicate refers to a set of individuals, this can be further spelled out as in (1c). But if we have a higher-order logic in which we can predicate things of properties and relations, we could also have defined the predicate-argument structure the other way around, as in (2a):

(2) Jane snores.
   a. Jane(Snore)
   b. \[ \{ \text{Jane(Snore)} \} = 1 \text{ iff } [\text{Snore}] \in [\text{Jane}] \]
   c. \[ \text{Jane} = \{ X \subseteq U | j \in X \} \]

If we take the predicate to snore to be an argument of the subject Jane, we are switching the predication relations around, and therefore the set-membership relation. Instead of checking whether Jane is a member of the denotation of to snore, (2a) claims that the property of snoring is one of the properties Jane has. In (2b) we formalize this as the requirement that the denotation of the VP be a member of the denotation of the proper name Jane in order for the sentence to be true. Of course the formula in (2a) is ill-formed in a first-order logic, so we have to use a second-order logic. In order to interpret the second-order formula in (2a), we cannot translate the proper name Jane as an individual constant, for properties cannot be related to individuals by set-membership relations. However, we can preserve the interpretation of predication as set-membership if we give the proper name Jane a translation in terms of sets of properties. After all, a property can be a member of a set of properties. Intuitively, the switch from individual constants to sets of properties is related to different perspectives on the individual. So far, we have looked at proper names as referential expressions: they point to a specific individual in the universe of discourse of the model. But we can also think of individuals as characterized by the properties they have. Suppose we know Jane as a person who has brown hair, who plays soccer, who likes black coffee, and who snores. Then the name Jane immediately gives rise to the picture of this individual having all these properties. This is what corresponds to the translation of the proper name Jane not as an individual constant, but as a bundle of properties (the properties that Jane has). Given that properties are extensionally

interpreted as sets of individuals, we claim that the denotation of Jane in (2b) is a set of sets, namely the set of all those sets \( X \) such that \( j \) is a member of \( X \). This is spelled out in (2c), and it is reflected in figure 8.1.

![Figure 8.1: Jane snores](image)

The introduction of a set of sets is the standard translation of NPs in Montague grammar (cf. Montague 1973). Sets of sets are outside the scope of a first-order logic, so when we discussed predicate logic, we could not raise the possibility of such an interpretation. But once we have made higher-order interpretations available in our semantic theory, we can easily introduce (2c) along with (1c), because they are fully equivalent. This means that if we combine a proper name with a VP, we can choose to make the NP the semantic argument of the VP, or the VP the semantic argument of the NP. It does not matter which predicate-argument structure we adopt for the sentence, for the two options lead to different, but equivalent translations. The translations are equivalent, because they lead to the same model-theoretic interpretation which requires the individual Jane to be a member of the set of snoring individuals in order for the sentence to be true. We conclude from this that the introduction of a second-order interpretation does not contribute anything to the interpretation of proper names, but it does not do any harm either. For quantificational NPs, the situation is different, and it is here that we see the main advantage of the interpretation of NPs as sets of sets.

In chapter 4, we argued that first-order logic cannot treat quantificational NPs such as every student in the same way as proper nouns, because these expressions are non-referential in the sense that they do not refer to particular individuals in the domain. As a result, (3b) cannot be the correct translation of (3):

(3) Jane snores.
   a. Jane(Snore)
   b. \[ \{ \text{Jane(Snore)} \} = 1 \text{ iff } [\text{Snore}] \in [\text{Jane}] \]
   c. \[ \text{Jane} = \{ X \subseteq U | j \in X \} \]
Every student danced.

\begin{enumerate}
\item \[\lambda x \forall P \text{ every student } [x \in P \text{ danced }]
\item \textbf{wrong:} Dance(every student)
\item \[\forall x \text{ Student}(x) \rightarrow \text{Dance}(x)]
\end{enumerate}

The translation of (3) in first-order predicate logic is given in (3c), and it provides the correct truth conditions for the sentence. However, as pointed out in chapter 7, the drawback of this translation is that we lose the insight that the NP and VP are related via a predication relation. Although there are syntactic units representing both the NP and the VP in (3a), these syntactic constituents are not treated as semantic constituents in the translation (3c). We can restore compositionality if we assume that the predicate-argument structure is not as in (3b), but as in (4b):

\begin{enumerate}
\item Every student danced.
\item \[\lambda x \text{ every student } [x \in P \text{ danced }]
\item Every student(danced)
\item \[\lambda x \text{ every student } [x \in P \text{ danced }]
\item \[\lambda x \text{ every student } [x \in P \text{ danced }]
\item \[\lambda x \text{ every student } [x \in P \text{ danced }]
\item \[\lambda x \text{ every student } [x \in P \text{ danced }]
\item \[\lambda x \text{ every student } [x \in P \text{ danced }]
\item \[\lambda x \text{ every student } [x \in P \text{ danced }]
\item \[\lambda x \text{ every student } [x \in P \text{ danced }]
\end{enumerate}

In (4b), the NP is the predicate, and the VP its argument. This translation is the exact mirror image of the syntactic structure of the sentence given in (4a). If we interpret predication in terms of set-membership as before, a representation like (4b) leads us to say that the sentence is true if and only if the set of individuals denoted by the VP is a member of the interpretation of the NP as in (4c). In a second-order logic, the assumption that the NP denotes a set of properties allows us to interpret this statement in such a way that the property described by the VP is a member of the set of properties denoted by the NP. The intuition is that an NP like \textit{every student} denotes the set of properties every student has. In the context of (4), the establishment of a predication relation means that dancing is one of the properties every student has. Other properties in the set denoted by \textit{every student} may be to be intelligent, to prepare for exams, to like the arts, etc. In order to support the claim that to dance is a property of every student, we have to make sure that the set of dancers is a member of the set of sets which are a superset of the student set (4d). This interpretation is reflected in the Venn diagram in figure 8.2.

We can easily extend this interpretation model to other quantificational NPs. For instance the NP \textit{some students} denotes the set of properties that some students have. In the extensional view of properties as sets of individuals, this corresponds to the set of individuals that have a non-empty intersection with the set of students. This leads to the Venn diagram in figure 8.3. A sentence like (5) is true if and only if the property of laughing is a member of the set of properties some students have:

\begin{enumerate}
\item Some students laughed.
\end{enumerate}

The approach also generalizes to NPs that cannot be translated in first-order predicate logic, such as \textit{most cats}:

\begin{enumerate}
\item Most cats have green eyes.
\end{enumerate}
A sentence like (6) is true if and only if the intersection of the set of cats with the set of individuals that have green eyes contains more than half of the total number of individuals that make up the set of cats.

The translation of NPs in terms of sets of sets corresponds with the mathematical notion of a generalized quantifier. Accordingly, the study of the properties of expressions that denote families of sets is referred to as Generalized Quantifier theory. As mentioned earlier, an interpretation of NPs as denoting generalized quantifiers (sets of sets) is built into Montague grammar. Barwise and Cooper (1981) apply the insights of the mathematical theory of generalized quantifiers to natural language and work out the semantic properties of NPs in this framework. Their starting point is that a sentence of the form \([s \, NP \, VP]\) is true iff the denotation of the VP is a member of the generalized quantifier, that is, the family of sets denoted by the NP. We write \(A\) as the denotation of the common noun: \(A = \{N\}\), and \(|A|\) for the cardinality of the set \(A\). The truth conditions for various NPs can now be formulated as follows:

\[
\begin{align*}
\text{(7) a. } & [\text{Jane}] = \{X \subseteq U \mid j \in X\} \\
\text{b. } & [\text{all N}] = \{X \subseteq U \mid A \subseteq X\} \\
\text{c. } & [\text{some N}] = \{X \subseteq U \mid X \cap A \neq \emptyset\} \\
\text{d. } & [\text{no N}] = \{X \subseteq U \mid X \cap A = \emptyset\} \\
\text{e. } & [(\text{at least two}) \, \text{N}] = \{X \subseteq U \mid |X \cap A| \geq 2\} \\
\text{f. } & [(\text{most}) \, \text{N}] = \{X \subseteq U \mid |X \cap A| > 1/2 \cdot |A|\} \text{ or } \{X \subseteq U \mid |X \cap A| > |X - A|\}
\end{align*}
\]

A second-order interpretation of determiners as denoting relations between sets solves the problems related to the interpretation of quantifiers like most. The examples show that the translation of NPs as sets of properties is a fruitful way to develop a compositional interpretation of NPs and to generalize over NPs of different kinds (proper names, quantificational NPs definable in first-order logic, quantificalional NPs not definable in first-order logic).

Barwise and Cooper (1981) emphasize that it is the NP which corresponds to the generalized quantifier, not the determiner. But if we want to extract the contribution of the determiner from the formulas in (7), we can try ‘unpack’ the definitions. As Barwise and Cooper point out, determiners are functions that map common noun denotations onto generalized quantifiers. The generalized quantifier in turn takes the VP as its argument to build a proposition. This is illustrated for (4) with the tree-like representation in figure 8.4.

We call this the functional perspective on generalized quantifiers, because it mirrors the syntactic structure of the sentence as closely as possible.

\[
\begin{align*}
\text{(8) a. } & [\text{All} \,(A, B)] = 1 \text{ iff } A \subseteq B \\
\text{b. } & [\text{Some} \,(A, B)] = 1 \text{ iff } A \cap B \neq \emptyset \\
\text{c. } & [\text{No} \,(A, B)] = 1 \text{ iff } A \cap B = \emptyset \\
\text{d. } & [(\text{at least five}) \,(A, B)] = 1 \text{ iff } |A \cap B| \geq 5 \\
\text{e. } & [(\text{most}) \,(A, B)] = 1 \text{ iff } |A \cap B| > |A - B| \\
\text{f. } & [(\text{many}) \,(A, B)] = 1 \text{ iff } |A \cap B| \geq m \\
\text{g. } & [(\text{many}) \,(A, B)] = 1 \text{ iff } |A \cap B| > m/n \cdot |A| \\
\end{align*}
\]

However, ‘undoing’ in a sense the effects of hierarchical structure and taking a set-theoretic perspective, we can view the denotation of a determiner as a binary relation between sets of individuals. This relational perspective on determiners is not in conflict with the functional view. Remember that two-place predicates like to love, to hit, etc. denote sets of individuals. There is an asymmetry between subjects and objects, because the predicate combines with its arguments one at a time (the object first, and then the subject). But any two-place predicate denotes a relation between individuals. In the same way, a determiner is interpreted as a two-place predicate, but a second-order one. There is an asymmetry between the common noun and the VP, because the determiner combines first with the common noun and then with the VP. However, any determiner denotes a set of pairs of sets, namely those sets which stand to each other in the relation denoted by the determiner. The semantic difference between the different determiners of a natural language like English resides in the specific relation they denote. For instance every denotes the subset relation, some the relation of non-empty intersection, no the relation of empty intersection, etc.

The relational perspective is adopted in the work of Zwarts (1983) and van Benthem (1986). As mentioned above, in Barwise and Cooper’s terminology, it is NPs that correspond to generalized quantifiers, not determiners. However, researchers who are working in the relational perspective often talk loosely about determiners as generalized quantifiers. As long as the context makes it clear what type of denotation is intended by the term ‘generalized quantifier’, this is rather innocent. Here are some examples of how determiners are interpreted in the relational perspective:
The variables $m$ and $n$ in (8) stand for natural numbers used to calculate the relations between the sets. These numbers are generally context-dependent and assumed to be known to the language users in a given situation. Determiners like *many* and *few* can have either a cardinal or a proportional interpretation, as illustrated in (9):

(9) a. There are many smart students in the semantics class.
    b. Many students in the semantics class are really smart.

(9a) means that the number of smart students in the semantics class is rather high. This is captured by the cardinal interpretation of *many* in (8f). (9b) claims that among the students of the semantics class, many of them are really smart. Given that we now calculate the number of smart students with respect to the set of students in the semantics class as a whole, we have a proportional interpretation, which is captured by (8g).

Note that (9b) but not (9a) is compatible with the number of students in the semantics class being small. The difference between cardinal and proportional interpretations is reflected in the different truth conditions of (8f) and (8g). They illustrate one advantage of the relational, set-theoretic perspective, namely the fact that it allows us to focus on the properties of the determiner, rather than the NP as a whole. The set-theoretical insight that determiners denote relations between sets of individuals allows us to apply methods from the theory of relations to the semantics of natural language determiners. There are two types of properties of determiners that are interesting to study in this perspective. On the one hand, we want to find out what properties natural language determiner denotations have in common and how to define these characteristics in the general theory of relations (section 8.2). On the other hand, we want to see if there are semantic properties which characterize certain subclasses of determiners, and investigate the role these properties play in accounting for the distribution of generalized quantifiers in various contexts (section 8.3).

### 8.2 Constraints on determiner denotations

The reason that *most* cannot be represented in first-order logic is that the determiner is essentially dependent on the interpretation of the head noun. The well-known quantifiers \( \forall \) and \( \exists \) from first-order logic range over the entire domain of discourse and cannot provide an interpretation for such determiners as *most*. However, the interpretation of determiners as (second-order) relations between sets allows a unified interpretation of all natural language determiners, because this approach takes into account the contribution of the common noun. As Barwise and Cooper (1981) put it, natural language determiners "live on" the denotation of the common noun. Another way of putting is to say that determiners in natural language tend to be *conservative*. Consider (3) again, repeated here as (10):

(10) Every student danced.

The evaluation of (10) requires us to check for each student whether or not (s)he has the dancing property. We are not concerned with dancing individuals outside the set of students, for they do not have an impact on the truth value of (10). The determiner *every* is 'left leaning' in the sense that we look first at the set of individuals that belong to the common noun denotation, and determine which of those satisfy the VP-denotation. We do not take into consideration individuals that have the property denoted by the VP, but that are not in the denotation of the common noun. Formally, conservativity is defined as follows:

- **Conservativity**
  
  For all \( A, B \subseteq U \): \( Q_U(A, B) \rightleftharpoons Q_U(A, A \cap B) \)

The conservativity of natural language determiners is reflected in the equivalence between pairs of sentences like the ones in (11):

(11) a. All children sing \( \equiv \) All children are children that sing
    b. Most children sing \( \equiv \) Most children are children that sing
    c. No children sing \( \equiv \) No children are children that sing

Not all natural language determiners are conservative. Notable exceptions are *only* and certain interpretations of *many* (cf. Westerstål 1985, de Hoop and Solà 1996, Herburger 1997):

(12) a. Only babies cry \( \not\equiv \) Only babies are babies that cry
    b. Many Scandinavians have won the Nobel Prize \( \not\equiv \) Many Scandinavians are Scandinavians who have won the Nobel Prize
    c. Many incompetent cooks applied \( \not\equiv \) Many incompetent cooks are cooks that applied

Instead of 'left leaning' determiners, we seem to have 'right leaning' determiners in these cases: (12a) is equivalent to 'The only crying individuals are crying babies' and (12b) reads as: 'Many individuals who won
the Nobel Prize are Scandinavians who won the Nobel Prize'. To a certain degree, one can argue that these cases are exceptional. *Many* has conservative interpretations besides the non-conservative one illustrated in (12b), so maybe this is just a special use of the determiner. Furthermore, it has been argued that *only* is not in fact a determiner, because it can also modify NPs as in *only John or only some babies* and even VPs as in *(John only) swims*. Therefore, the property of Conservativity is considered to be a (nearly) universal property of natural language quantifiers.

Another property which is a quasi-universal characteristic of natural language quantifiers is Extension. Extension states the principle of context neutrality. It guarantees that the determiner has the same structure in every model. Extension is formulated as closure under growth of the universe of discourse:

- **Extension**
  
  For all $A, B \subseteq U$: if $Q_U(A, B)$ and $U \subseteq U'$ then $Q_U(A, B)$

This means that a quantifier which has a certain meaning as a relation between two sets $A$ and $B$ will not change its meaning if the universe becomes bigger without affecting the size of the sets $A$ and $B$. An example of a non-extensional quantifier is provided by certain uses of *many*, as discussed by Westerståhl (1985) and illustrated by (13):

\[(13)\quad \text{There are many native speakers of Dutch.}\]

(13) can be a true statement if a speaker in the European parliament tries to make the point that all proceedings need to be translated into Dutch. But given that Holland is a small country, this statement will prove false when a speaker is trying to make the same point in a meeting of the United Nations. From one context to the next, the number of native speakers of Dutch does not change, but the size of the universe of discourse increases dramatically. If the number of individuals that counts as ‘many’ is a function of the number of positive instances compared to the size of the universe of discourse of the model, we can account for the different truth values of (13) in different contexts. *Many* is thus an example of a strongly context-dependent determiner, which may even lose the property of extension in some of its uses.

The combination of Conservativity and Extension leads to a strong version of conservativity, dubbed Conservativity$^+$ by van Benthem (1986):

- **Conservativity$^+$**
  
  For all $A, B \subseteq U$: $Q_U(A, B) \iff Q_A(A, A \cap B)$

Extension guarantees that we do not need to take into consideration individuals which are not in $A$ or $B$, for the meaning of the determiner is independent of the size of the universe. Conservativity tells us that individuals that are in $B$, but not in $A$, do not play a role in determining the truth value of the proposition, because the determiner is ‘left leaning’. Strong conservativity thus effectively restricts the universe of discourse to the left-hand argument. Neither elements in $B - A$, nor individuals in $U - (A \cup B)$ are taken into account when the sentence is evaluated.

Another useful property of determiners is *Quantity*. Quantitative determiners are insensitive to the individual characteristics of the members of $A$ and $B$. The truth value of the sentence depends only on the number of elements in $A$ and $A \cap B$. Quantity is formally defined as closure under permutation of individuals:

- **Quantity**
  
  For all $A, B \subseteq U$ and all permutations $m$ of $U$:
  
  $Q_U(A, B) \iff Q_U(m[A], m[B])$

A permutation $m$ is an operation which maps a set $X$ onto another set $X'$ by relating every member of $x$ with some member of $X'$. $X'$ has the same number of elements as $X$, because permutation affects only the identity of the members. Closure under permutations means that the quantifier does not care which individuals are in $A$ and $B$, as long as the same number of individuals remains in each set. Most well-known determiners are quantitative in nature, for example *some* in (14a):

\[(14)\quad \begin{align*}
  a. & \quad \text{Some students are lazy.} \\
  b. & \quad \text{Mary's bike has been stolen.}
\end{align*}\]

(14a) is true if and only if the set of lazy students is not empty. It does not matter which students are lazy, but there must be at least some. We can mix up the set of students and the set of lazy individuals without any change in truth value as long as the intersection remains non-empty. Not all determiners are quantitative. Possessive determiners such as the one in (14b) are not closed under permutation. For (14b) to be true it is not enough that there be at least one bike which has been stolen, it has to be Mary’s bike. We cannot apply just any permutation, because the determiner is sensitive to a specific property of the individual bike, namely that it is owned by Mary. So possessives are not quantitative determiners.

The relational interpretation of quantified NPs can be pictured in a Venn diagram. The combination of the three properties of Conservativity, Extension and Quantity makes the interpretation of the determiner only
dependent on the cardinality of \( A - B \) and \( A \cap B \), that is \( a \) and \( b \) in the Venn diagram in figure 8.5.

The Venn diagram shows that the combination of the three general constraints greatly simplifies the interpretation of the determiner, because we can leave out two of the four subsets of the universe \( U \) as irrelevant, and for the two remaining sets we only need to know the number of elements in these sets. This effectively reduces the contribution of determiners that satisfy the three general constraints to a relation between two numbers: the number of individuals in the set \( A \) and the number of individuals in the set \( A \cap B \).

8.3 Subclasses of determiners

Certain properties do not characterize natural language quantifiers in general, but are particular to subclasses of quantifiers, such as the weak/strong distinction, definiteness and monotonicity.

8.3.1 The weak/strong distinction

The weak/strong distinction plays a role in the interpretation of existential \textit{there}-sentences and related constructions, as observed by Milsark (1977), Barwise and Cooper (1981), de Jong (1987) and Keenan (1987). Consider:

\[\begin{align*}
\text{(15)} & \quad \text{a. There is a cat in the garden.} \\
& \quad \text{b. *There is the cat in the garden.} \\
& \quad \text{c. There are some/two/many/no cats in the garden.} \\
& \quad \text{d. *There is every/neither cat in the garden.} \\
& \quad \text{e. *There are most/not all/both cats in the garden.}
\end{align*}\]

The contrast between the (a-) and (b-)sentences of (15)–(16) suggests that indefinite NPs are admitted in existential sentences because the function of such constructions is to predicate existence. As pointed out in section 5.4.2, indefinite NPs introduce new individuals into the universe of discourse, which then become available for further reference. (17b) is not downright ungrammatical, but it gets a very different interpretation from (17a). (17a) is typically used to measure the distance we covered, whereas (17b) can only be used if we somehow have a particular mile in mind and assert that we walked (rather than biked, or drove) that particular mile. The measuring interpretation is subject to the same restrictions as the existential contexts in (15) and (16). For definite NPs, existence is presupposed, so they are not felicitous in constructions which introduce new individuals. Milsark (1977) points out that this distinction needs to be further generalized in view of examples (c)–(e) of (15)–(17). It is not so clear what is indefinite about \textit{no}, for instance, which would allow it to occur in existential constructions whereas \textit{neither} is not allowed. Similarly, it is hard to explain what is definite about \textit{most} which blocks it from existential contexts, whereas \textit{many} is possible. Therefore Milsark replaces the definite/indefinite contrast with a more general distinction between weak and strong NPs. The weak/strong distinction plays an important role in the study of syntactic and semantic properties of indefinite NPs, for instance in the characterization of generic and partitive interpretations of indefinites in a generalized quantifier perspective (see de Hoop 1992 for discussion).

One way to determine the strength of a determiner is the following test, proposed by Barwise and Cooper (1981):

\[\text{(18) Det N is a N/ are Ns}\]

According to Barwise and Cooper, a determiner is positive strong if this statement is a tautology in every model in which the quantifier is defined. A determiner is negative strong if it is a contradiction, and it is weak if the
truth of the statement depends on the model. This correctly classifies the following determiners:

(19) 
\[
\begin{align*}
&\text{a. Every cat is a cat.} & \text{[pos. strong]} \\
&\text{b. Both cats are cats.} & \text{[pos. strong]} \\
&\text{c. Not all cats are cats.} & \text{[neg. strong]} \\
&\text{d. Neither cat is a cat.} & \text{[neg. strong]} \\
&\text{e. Many cats are cats.} & \text{[weak]} \\
&\text{f. Ten cats are cats.} & \text{[weak]} \\
&\text{g. Some cats are cats.} & \text{[weak]} \\
&\text{h. No cats are cats.} & \text{[weak]} 
\end{align*}
\]

We have to be careful, for the judgments are not always very intuitive. (19a) and (c) are easy to classify, because the universal quantifier comes out true even on an empty domain, so the negation of the universal quantifier always comes out false. As a result, every is positive strong, and not all is negative strong.

The determiners in (19b) and (d) are positive and negative strong respectively, because they are definite (see section 8.3.2 below for a more precise definition of definiteness in generalized quantifier theory). Definite NPs are only defined when their presupposition is fulfilled. This means that (19b) and (d) are exclusively defined in models in which there are exactly two cats. Obviously, in such a model, (19b) is a tautology, and (d) a contradiction. The characterization of many in (19e) as a weak determiner is based on its behavior in models which contain less than 'many' cats (whatever the context-dependent number happens to be). In such a model the sentence comes out false, but of course in all other models it is true. If the sentence is sometimes true, sometimes false, it is neither a tautology nor a contradiction, so the determiner is classified as weak. (19f) is another example of a weak determiner. The sentence is true in a model which contains ten cats or more, false in a model in which there are less than ten cats. The argumentation for some and no in (19g) and (h) is similar, but may be somewhat harder to see. (19g) is always true, except in a model in which there are no cats, in which the sentence comes out false. Compare this with sentence (19h), which is always false, except when we evaluate it in models with no cats. Because of this one exceptional case, the determiners some and no count as weak.

We can use this to explain the distribution of determiners in existential sentences in the following way. Existential sentences are taken to predicate existence of the subject noun phrase. If we assume a predicate \( E \) for 'existence in the universe of discourse \( U \)', we can write the quantificational structure of existential sentences as \( Q(A, E) \). Because of Conservativity this statement is equivalent to \( Q(A, A \cap E) \), which is in turn equivalent to \( Q(A, A) \), because all sets are a subset of the set of individuals that exist in the universe of discourse. Notice that \( Q(A, A) \) is the quantificational structure of the sentences in (19). The claim is then that strong quantifiers are blocked from existential sentences because it would make them express a tautology or a contradiction.

There are two general problems with this line of explanation. One is that the definition of the weak/strong distinction proposed by Barwise and Cooper is not very intuitive. In particular, we have to accept that some and no are weak because of their behavior in models where the denotation of the common noun is the empty set. In some sense, this is not what we would like to take as the basis of our linguistic theory. Furthermore, natural language usually does not rule out tautologies and contradictions as ungrammatical, as illustrated by the well-formedness of a sentence like (20):

(20) There are less than zero cats in the garden.

There is no doubt about the contradictory character of (20): gardens do not contain negative numbers of cats. Even so, the sentence is fully grammatical (and understandable as a contradiction). Barwise and Cooper's characterization of the weak/strong distinction has been criticized for these reasons, but the general insight that there is a semantic difference between weak and strong determiners which explains their distribution in existential sentences has been preserved. Keenan (1987) exploits the property of symmetry to define a group of determiners as existential. Symmetry is one of the well-known properties of the theory of relations we studied in section 4.1.4. It has been observed that weak, but not strong determiners are symmetric. Some is symmetric, because we know that if some linguists are Spanish, then some Spanish are linguists. No is symmetric, for if no Stanford student is from Holland, then no one from Holland is a student at Stanford. Similarly, we can determine that at least five, at most five, many and few (in their cardinal interpretation) are symmetric. In combination with conservativity, the property of symmetry implies that only the intersection of \( A \) and \( B \) determines the truth value of the sentence:

\[
\begin{align*}
Q(A, B) &\iff Q(B, A) & \text{(symmetry)} \\
Q(A, B) &\iff Q(A, A \cap B) & \text{(conservativity)} \\
Q(B, A) &\iff Q(B, B \cap A) & \text{(conservativity)} \\
\text{Thus:} & \\
Q(A, B) &\iff Q(A, A \cap B) \iff Q(B, B \cap A) \\
\text{As a result:} & \\
Q(A, B) &\iff Q(A \cap B, A \cap B)
\end{align*}
\]
The equivalencies in (21) show that determiners that are both conservative and symmetric are such that only the number of elements in the intersection of A and B plays a role in determining the truth value of the sentence. This can help us explain the constraints on existential sentences in the following way. We assume that in existential sentences the first argument of the quantifier is E, the predicate ‘exist’, which is satisfied by every individual in the universe of discourse U. Taking the part cats in the garden in (16) to denote the intersection of the set of cats and the set of individuals in the garden, we can write the quantificational structure of existential sentences as \( Q(E, A \cap B) \). For conservative and symmetric quantifiers, this is equivalent to \( Q(A, B) \), according to the following reasoning:

\[
(22) \quad Q(E, A \cap B) \iff Q(E \cap A \cap B, E \cap A \cap B) \\
\text{(see above)}
\]

\[
Q(E \cap A \cap B, E \cap A \cap B) \iff Q(A \cap B, A \cap B) \\
(A, B \text{ are subsets of } E)
\]

\[
Q(A \cap B, A \cap B) \iff Q(A, B) \\
\text{(see above)}
\]

So we know that for symmetric quantifiers \( Q(E, A \cap B) \) is equivalent to \( Q(A, B) \). Keenan (1987) defines existential determiners as determiners which satisfy exactly this equivalence. Existential determiners are the only determiners which allow an existential interpretation of there-constructions. Let us consider some examples. Because of the symmetry of the determiner, (23a) and (23b) are equivalent, and (23a) is appropriately characterized as an existential sentence. (23c) and (23d) are not equivalent, because every is not symmetric. Whatever interpretation we may be able to cook up for (23c), it will not qualify as an existential sentence:

\[
(23) \quad a. \text{ There are five cats in the garden.} \\
b. \text{ Five cats are in the garden.} \\
c. \text{ There is every cat in the garden.} \\
d. \text{ Every cat is in the garden.} \\
e. \text{ There are less than zero cats in the garden.} \\
f. \text{ Less than zero cats are in the garden.}
\]

Finally, we observe that (23f) is as contradictory as (23e), but the two sentences are clearly equivalent. This appropriately characterizes five and less than zero as weak, existential determiners, and every as a strong, non-existential quantifier. Keenan’s analysis correctly captures our intuitions about the meaning of the sentences in (23). It is also appropriate for cases such as (19), which do not involve ungrammaticality, but the unavailability of certain interpretations. It is harder to determine whether this purely semantic account also explains the lack of syntactic well-formedness of the (b), (d) and (e) examples in (15)–(16). The analysis explains why determiners that are not existential resist existential interpretations, but it does not explain why these determiners yield ungrammatical sentences. So there is more to be said about existential there-sentences.

The study of the distribution of NPs in existential contexts is a first illustration of the general approach in generalized quantifier theory. We study subclasses of determiners which have certain characteristics definable in terms of the theory of relations. On the basis of those properties and the universal constraints of conservativity, extension and quantity, we provide a semantic explanation of the behavior of certain classes of NPs.

### 8.3.2 Partitivity

We can explain the general constraints on partitive noun phrases of the form ‘NP of NP’ in similar ways. The data in (24) suggest that the NP embedded under the preposition of must be definite:

\[
(24) \quad a. \text{ Some of the students brought a gift.} \\
b. \text{ Each of these two linguists has a theory about partitives.} \\
c. \text{ The children ate all of the cake.} \\
d. \text{ One of them must have committed the murder.} \\
e. *\text{Some of all students brought a gift.} \\
f. *\text{Each of most linguists has a theory about partitives.} \\
g. *\text{The children ate all of no cake.}
\]

The issue of which NPs can appear in the partitive construction bears on the meaning of the NP as a whole. Furthermore, we observe that the partitive and non-partitive members of the pairs in (25) are quite similar in interpretation, the main difference being that in the case of partitives, the quantification appears to be over some specific, non-empty, contextually fixed set, here a particular set of students:

\[
(25) \quad a. \text{ some of the students} \\
b. \text{ few of the students} \\
c. \text{ none of the students}
\]
In cases where both constructions are felicitous, the main difference between partitive and non-partitive NPs resides in the status of the set of individuals quantified over. In order to account for this intuition, Barwise and Cooper (1981) propose an interpretation of the partitive use of the preposition of as an operator which maps NP-denotations (sets of sets) onto CN-denotations (sets) by taking the intersection of all the sets in the generalized quantifier. Once of has applied, we have obtained a set again, which any determiner can combine with. There is a further restriction though, namely that the partitive construction is felicitous only in case the intersection of all the sets in the generalized quantifier denotation is a subset of the set of individuals denoted by the common noun. This requirement translates as the following claim: ∩[NP] ⊆ [CN]. Interestingly, the NPs which satisfy this constraint in a non-trivial way (i.e. in such a way that the intersection of all sets in the NP is not the empty set) are exactly those which satisfy Barwise and Cooper’s definition of definite NPs. According to Barwise and Cooper, an NP is definite iff it has a non-empty set B which is a subset of every set in the denotation of the generalized quantifier. This set B is called the generator of the definite NP. It is clear from the diagram in figure 8.6 that the generator of a definite NP such as the students is the set of all students.

![Diagram](D)

Figure 8.6: The students danced

The comparison with figure 8.3 above illustrates that an indefinite NP like some students does not have a unique generator set.

Note that the constraint on partitive NPs is stricter than strength. An NP like most students is strong, but it does not have a unique generator set. Universally quantified NPs like every student do have a unique generator set, namely the empty set, which, by definition is a subset of every set. Given that the generator set of a definite NP should be a non-empty set, universally quantified NPs do not satisfy the definition of definite NPs, and they do not satisfy the partitivity constraint in a non-trivial way. According to Barwise and Cooper, it is the ability of definite NPs to uniquely determine the generator set that allows the NP to play the role of a common noun and recombine with a determiner. The additional information that is supplied by the definite determiner is just that the set being quantified over is non-empty. This accounts for the intuition that in partitive constructions we quantify over some specific, contextually determined set of individuals (e.g. the set of students in 25). Further restrictions are necessary in order to explain why (26a) is good, whereas (26b) is not:

(26) a. One of the two books was sold out.
   b. *One of both books was sold out.

Ladusaw (1982) points out that the main difference between these two NPs is that the two N has a group-level denotation, whereas both N does not. This is supported by the data in (27), which involve a group-level predicate:

(27) a. The two students are a happy couple.
   b. *Both students are a happy couple.

A property like to be a happy couple is typically not ascribed to individuals, but to groups of individuals. The ungrammaticality of (27b) comes about as a result of the semantic nature of both N. This NP requires a distributive interpretation in which each of the two individuals has the property denoted by the VP. A property like to be a happy couple is typically not ascribed to individuals, but to groups of individuals. The relevance of a restriction to group-denoting expressions also explains some apparent counterexamples to the partitive constraint as in (28):

(28) a. John was one of several students who arrived late.
   b. That book could belong to one of three people.

The sentences in (28) are felicitous only in contexts in which the speaker has a particular group of individuals in mind. The capacity of indefinite NPs to set up discourse referents available for further reference plays a role here (compare our informal discussion of discourse anaphora in chapter 5). It would lead too far to discuss all the intricate details of the partitive construction here, so the reader is referred to de Jong (1987) and de Hoop (1997) for further discussion.

### 8.3.3 Monotonicity

Monotonicity involves the possibility of inference to supersets or subsets of the set under consideration. As usual, we are interested in entailment relations, because of their impact on valid reasoning patterns in which the truth of some statement (the conclusion) follows from the truth of one or
more other statements (the premises). This motivates a general interest in inference patterns.

Following Barwise and Cooper (1981) and Zwarts (1981, 1986), we define upward monotonicity as follows:

• A quantifier \( Q \) is monotone increasing (\( \text{mon} \uparrow \)) if:
  \[ Q(A, B) \text{ and } B \subseteq B' \text{ implies } Q(A, B') \]

Some examples:

(29) \( \text{Mon} \uparrow \) quantifiers

a. All children came home late \( \rightarrow \) All children came home
b. Most children came home late \( \rightarrow \) Most children came home
c. At least five children came home late \( \rightarrow \) At least five children came home

Not all quantifiers are monotone increasing. For instance, the inferences do not go through in the following cases:

(30) a. No children came home late \( \not\rightarrow \) No children came home
b. Not all children came home late \( \not\rightarrow \) Not all children came home
c. Less than five children came home late \( \not\rightarrow \) Less than five children came home

Some of these quantifiers have the related property of downward monotonicity:

• A quantifier \( Q \) is monotone decreasing (\( \text{mon} \downarrow \)) if:
  \[ Q(A, B) \text{ and } B' \subseteq B \text{ implies } Q(A, B') \]

Some examples:

(31) \( \text{Mon} \downarrow \) quantifiers

a. No children came home \( \rightarrow \) No children came home late
b. Not all children came home \( \rightarrow \) Not all children came home late
c. At most five children came home \( \rightarrow \) At most five children came home late

So monotonicity properties typically tell you that you can make inferences towards larger or smaller sets. In other words, they correspond to closure under superset or (finite) subset formation, respectively. Not all quantifiers are either monotone increasing or monotone decreasing. Some non-monotonic quantifiers are the following:

(32) Non-monotonic quantifiers

a. Exactly two children came home late \( \not\rightarrow \) Exactly two children came home
b. An even number of children came home late \( \not\rightarrow \) An even number of children came home
c. More than two but less than five children came home late \( \not\rightarrow \) More than two but less than five children came home

One of the contexts in which the property of monotonicity plays a role is in the explanation of conjunction and disjunction reduction, as argued by Zwarts (1981, 1986). Consider the sentences in (33):

(33) a. Sara sang and danced.
b. Sara sang and Sara danced.
  c. Sara sang or danced.
d. Sara sang or Sara danced.

For transformational grammarians in the ’60s and ’70s, the relations between reduced and unreduced sentences were a puzzle. At first sight, it looks attractive to transformationally derive (33a) from (33b). After all, the two sentences are equivalent. Similarly, we could try to derive (33c) from (33d) on the basis of the equivalence between the two sentences. However, it turns out that not every sentence of the form (34a) is equivalent to one of the form (34b):

(34) a. NP VP\(_1\) and/or VP\(_2\)
b. NP VP\(_1\) and/or NP VP\(_2\)

As far as quantified NPs are concerned, the equivalence works in examples like (35a). It works in (35b) if we switch or into and. It reduces to a one-way implication in cases like (35c), (d) and (e), and it breaks down entirely in cases like (35f):
a. Every student sang and danced ↔
   Every student sang and every student danced
b. No student sang or danced ↔
   No student sang and no student danced
c. At least two students sang and danced →
   At least two students sang and at least two students danced
d. At most two students sang and danced ↔
   At most two students sang and at most two students danced
e. At most two students sang or danced →
   At most two students sang and at most two students danced
f. Exactly two students sang and danced ≠
   Exactly two students sang and exactly two students danced

The observation that the sentences in (35) share the same syntactic structure, but participate in different inference patterns supports the claim that a purely syntactic approach to conjunction reduction is not the right line of explanation. If the only difference between the sentences in (35) is the choice of the NP, it is likely that conjunction and disjunction reduction are somehow dependent on the semantic properties of the NP. The property of monotonicity springs to mind, because of the similarities between entailment relations (defined on propositions) and inference to subsets and supersets (defined on sets). In chapter 3, we discussed entailment relations between conjoined/disjoined statements and the atomic propositions making up the complex propositions. These entailment relations have set-theoretical counterparts we can appeal to in our treatment of the examples in (35). If we assume that the conjunction of two VPs is interpreted as intersection of the two sets involved, and disjunction of two VPs is interpreted as set union, we obtain the following statements about relevant subset and superset relations:

- \([\text{VP}_1 \text{ and } \text{VP}_2] \subseteq [\text{VP}_1]\) and
  \([\text{VP}_1 \text{ and } \text{VP}_2] \subseteq [\text{VP}_2]\) because
- \([\text{VP}_1 \cap \text{VP}_2] \subseteq [\text{VP}_1]\) and
  \([\text{VP}_1 \cap \text{VP}_2] \subseteq [\text{VP}_2]\)

- \([\text{VP}_1] \subseteq [\text{VP}_1 \text{ or } \text{VP}_2]\) and
  \([\text{VP}_2] \subseteq [\text{VP}_1 \text{ or } \text{VP}_2]\) because
- \([\text{VP}_1] \subseteq [\text{VP}_1] \cup [\text{VP}_2]\) and
  \([\text{VP}_1] \subseteq [\text{VP}_1] \cup [\text{VP}_2]\)

We know that monotone increasing quantifiers are closed under superset formation, whereas monotone decreasing quantifiers are closed under subset formation. These properties are related to conjunction/disjunction reduction in a straightforward way. If a quantifier is monotone increasing and a conjunction of two properties is a member of the quantifier, each of these properties will be a member of the quantifier. After all, the conjunction is a subset of both, and the quantifier licenses the inference to a larger set. For the same reason we expect the disjunction of two properties to be a member of a monotone increasing quantifier if either one of the properties is. For monotone decreasing quantifiers, the inferences go the other way around, because they license inferences to smaller sets:

- \(\text{Mon} \uparrow \text{quantifiers}\)
  a. \(Q(A, B \cap C) \rightarrow Q(A, B) \quad \text{and} \quad Q(A, C)\)
  b. \(Q(A, B) \quad \text{or} \quad Q(A, C) \rightarrow Q(A, B \cup C)\)

- \(\text{Mon} \downarrow \text{quantifiers}\)
  a. \(Q(A, B) \text{ and } Q(A, C) \rightarrow Q(A, B \cap C)\)
  b. \(Q(A, B \cup C) \rightarrow Q(A, B) \text{ and } Q(A, C)\)

The patterns in (36) and (37) explain the one-way inferences in (35c), (35d) and (35e). They also rule out any implication relation between the two sentences in (35f): non-monotonic quantifiers do not allow this kind of inference pattern. Because there is no inference relating one sentence to the other, we cannot derive one sentence from the other by means of conjunction reduction. The general properties of monotone increasing and monotone decreasing quantifiers also explain one half of the inferences in (35a) and (35b). The inference from left to right in (35a) follows from the interpretation of conjunction as set intersection, just like in (35c). The inference from right to left in (35b) follows from the definition of disjunction as set union, just like in (35d).

In order to explain the equivalences in (33), (35a) and (35b), we still need to account for the other half of the inferences. This requires a more refined semantic analysis of the quantifiers involved. It turns out that we can define several linguistically interesting subsets of monotone increasing and decreasing quantifiers. The inference patterns in (36) and (37) follow directly from the monotonicity properties of the quantifiers involved. However, they cannot replace the original definition of monotone increasing and decreasing quantifiers given above, for they impose a weaker constraint on the quantifier denotation. In particular, we find that a subset of the set of mon \(
\uparrow \text{quantifiers} \) also licenses the inferences in (37). Such quantifiers are called filters. Furthermore, a subset of the set of mon \(
\downarrow \text{quantifiers} \) licenses...
the inferences in (36). They are called ideals. The formal definition of filters and ideals is as follows:

- filters
  \[ X \in Q \text{ and } Y \in Q \iff (X \cap Y) \in Q \]

- ideals
  \[ X \in Q \text{ and } Y \in Q \iff (X \cup Y) \in Q \]

Natural language expressions which have filter structure are universal quantifiers, definite NPs and proper names, as we can see from the following equivalences, repeated from (33a) and (35a):

(38) a. Every student sang and danced \iff
d\quad Every student sang and every student danced
d\quad Sara sang and danced \iff
d\quad Sara sang and Sara danced

Ideals in natural language are negative universal quantifiers like no (35b) and the negation of definite NPs and proper names. The full range of equivalences and one-way implications in (35) is now accounted for. The equivalence in (33b) requires a further refinement, for this pattern is valid for a subset of the set of filters only. Compare (33b), repeated here as (39a) with the one-way inference in (39b):

(39) a. Sara sang or danced \iff
d\quad Sara sang or Sara danced
d\quad Every student sang or danced \iff
d\quad Every student sang or every student danced

The inference from right to left is licensed by the monotone increasing nature of the quantifier. The inference from left to right is licensed only for ultrafilters. Examples of ultrafilters are proper names and definite descriptions. The formal definition of an ultrafilter is as follows:

- ultrafilters
  \[ X \in Q \text{ or } Y \in Q \iff (X \cup Y) \in Q \]

The examples discussed in this section show that conjunction and disjunction reduction can fruitfully be explained if we take a semantic approach to natural language and describe the properties of determiners in a mathematically precise way.

8.3.4 Negative polarity

Another application of the property of monotonicity to natural language semantics is the treatment of the phenomenon of negative polarity items. Negative polarity items are expressions in natural language which only occur in contexts with a negative flavor, compare:

(40) a. Susan did not say anything.
b. *Susan said anything.
c. I do not think that John has ever read a paper in philosophy.
d. *I think that John has ever read a paper in philosophy.
e. Jim did not lift a finger to help us.
f. *Jim lifted a finger to help us.

At first sight, one might say that negative polarity items are expressions that need to occur in a ‘negative’ environment. Negation can be provided by sentence negation, but it could also be some negative quantifier like nobody or never. Compare:

(41) a. Susan never says anything.
b. *Susan often says anything.
c. No student in linguistics ever read a paper in philosophy.
d. *Every student in linguistics ever read a paper in philosophy
e. Nobody lifted a finger to help us.
f. *Everybody lifted a finger to help us.

We would have to stretch our use of negation even further to account for the following contrasts:

(42) a. Susan seldom says anything.
b. *Susan always says anything.
c. At most five students in linguistics ever read a paper in philosophy.
d. *At least five students in linguistics ever read a paper in philosophy.
e. Few people lifted a finger to help us.
f. *Many people lifted a finger to help us.

We would have to explain why at least five, often and many are ‘positive’ and at most five, few and seldom are ‘negative’ rather than the other way round. We could try to reanalyze few as ‘not many’ and seldom as ‘not often’, but that would be begging the question why many is not reanalyzed as ‘not few’, and often is not reanalyzed as ‘not seldom’. Also, the fact
that one can say things like (43) suggests that many and few, and often and seldom are not exactly each other's negation:

(43)  a. There are not many majors in linguistics, but not few either.
   b. This child does not pray often, but not seldom either.

This argumentation is reminiscent of the discussion of the distribution of NPs in existential contexts. Intuitively, existential contexts are sensitive to the definite/indefinite distinction, but in order to account for a broader range of determiners, we generalized this to a notion of weakness and strength. As far as negative polarity items are concerned, there is an intuitive explanation in terms of negation, but it needs to be generalized in order to account for all the relevant cases. One line of explanation which has been explored by Ladusaw (1979), Zwarts (1986), van der Wouden (1997) and others is to assume that the constraint on negative polarity items is to be formulated in terms of monotonicity rather than strict negation. More specifically, the claim is that negative polarity items are restricted to downward entailing contexts. Obviously, negation is a downward entailing environment, but the definition includes a large number of other licensing contexts as well. The restriction to downward entailing contexts provides an interesting semantic explanation of the distribution of negative polarity items in (41)–(43). Compare Ladusaw (1996) and references therein for more discussion of syntactic, semantic and pragmatic aspects of negative polarity.

So far we have only defined monotonicity for the right argument of the quantifier (the VP-argument). Examples like those in (44) show that we also need to have access to the monotonicity properties of the determiner with respect to the first argument of the quantifier (the nomin al argument):

(44)  a. Every student who has ever studied logic takes a semantics class.
   b. No student who has ever studied logic fails to take a semantics class.

As argued above, every is a monotone increasing quantifier. It can easily be shown that this is true for the rightward argument only. In its left argument every is monotone decreasing:

(45)  a. Every student left for spring break →
     Every linguistics student left for spring break
   b. No student left for spring break →
     No linguistics student left for spring break

The generalization of the definitions of right monotonicity to left monotonicity is straightforward:

- A quantifier Q is left monotone increasing (↑ mon) if:
  Q(A,B) and A ⊆ A' implies Q(A',B)
- A quantifier Q is left monotone decreasing (↓ mon) if:
  Q(A,B) and A' ⊆ A implies Q(A',B)

As expected, and shown in (44), left monotone decreasing quantifiers license negative polarity items in their first argument.

In this chapter, we have seen that the theory of generalized quantifiers provides insight into a wide range of natural language quantifiers in different constructions. Furthermore, the theory can be extended in various ways. The theory does not only apply to determiners, but can account for quantification in the adverbial or verbal domain as well (compare Bach, Kratzer and Partee eds. 1993). De Swart (1991) takes adverbs of quantification like sometimes, always, never, often to denote relations between sets of events, and studies their semantic properties in a generalized quantifier perspective. Similarly, Hoeksema (1983) and others discuss monotonicity properties of comparative constructions, and Sánchez Valencia, van der Wouden and Zwarts (1994) use monotonicity patterns to study the semantics of temporal connectives like since and before.


### 8.4 Exercises

1. Show in a model with four entities that John's books is not a quantitative determiner whereas every book is.

2. The general constraints on determiner denotations (conservativity, extension and quantity) allow us to represent the space of possible determiner concepts in natural language graphically by means of a tree of numbers. Let A and B be sets, let a = |A − B|, and let b = |A ∩ B|. Then we can construct a tree of numbers corresponding to the possible pairs (a, b):